REINFORCEMENT LEARNING: LEARNING TO LEARN

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AN ERA OF CHANGE

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A MOTIVATING GAME

Rules of the game

- There is a heap with 21 sticks
- Each player is allowed to pick 4 sticks at maximum
- A player is not allowed to repeat the previous move
- A player wins if the opponent cannot play

A MOTIVATING GAME

- The optimal strategy is simple:

zones with multiples of 5 are safe!

- Suppose we are allowed to pick 5 sticks at maximum. What is the optimal strategy now?

- No simple solution!

A MOTIVATING GAME

s	$z=1$	$z=2$	$z=3$	$z=4$	$z=5$
0	-1	-1	-1	-1	-1
1	-1	1	1	1	1
2	1	1	1	1	1
3	1	1	-1	1	1
4	1	1	1	-1	1
5	1	1	1	1	-1
6	1	1	-1	1	1
7	-1	-1	-1	-1	-1

A MOTIVATING GAME

- Recall:
  s = number of sticks in the heap,
  z = previous move.

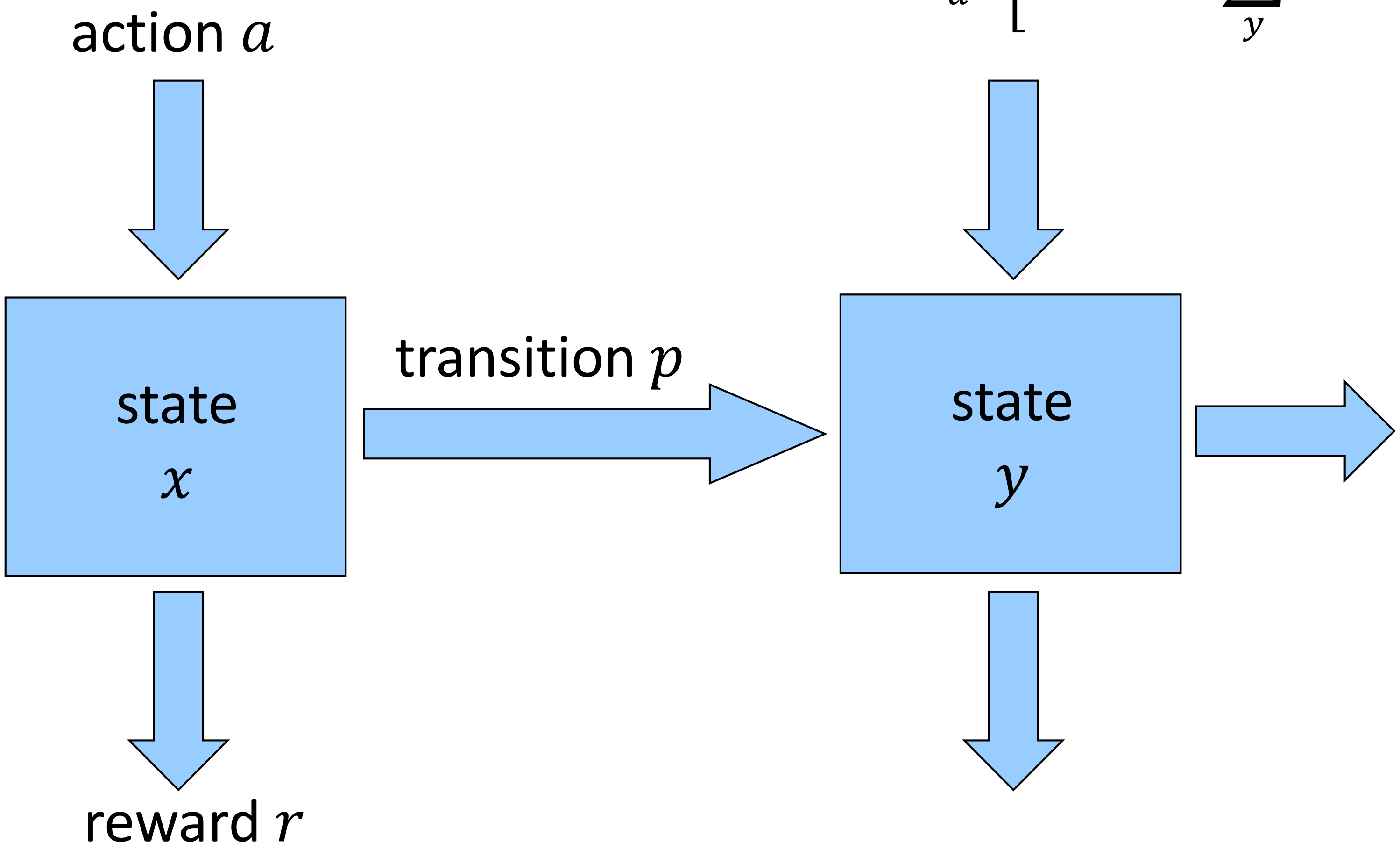
- Look at the rewards:
  $V(0,z) = -1$,
  $V(s,z) = 1$ for all $s < 0$ and $s + z \geq 0$.

- Dynamic programming:

$$V(s,z) = \max_{a \neq z}(\min_{b \neq a}[V(s-a-b,b)])$$

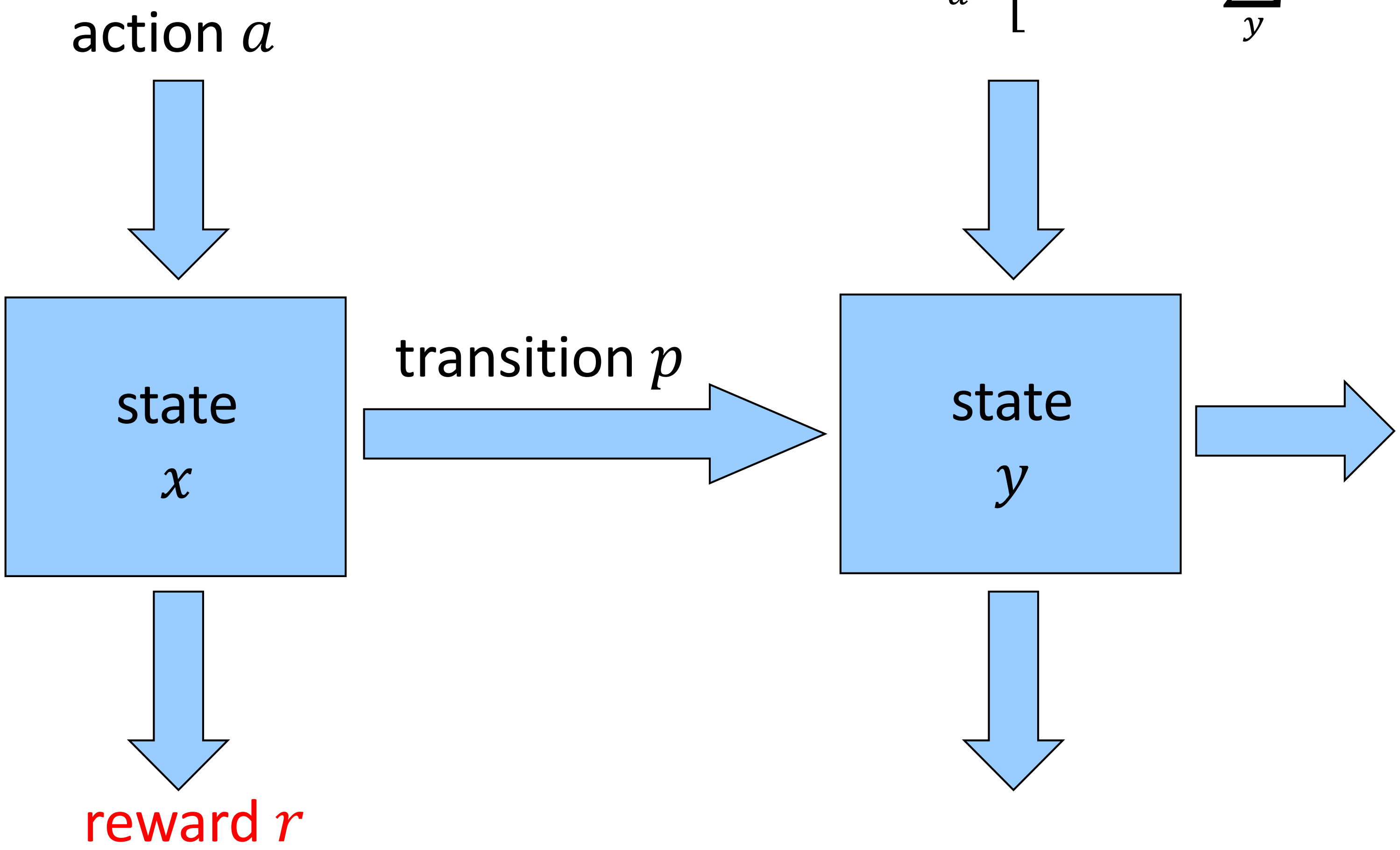
MARKOV DECISION PROCESSES

$$V(x) = \max_a \left[r(x,a) + \sum_y p(x,a,y) V(y) \right]$$



THE IOWA GAMBLING TASK

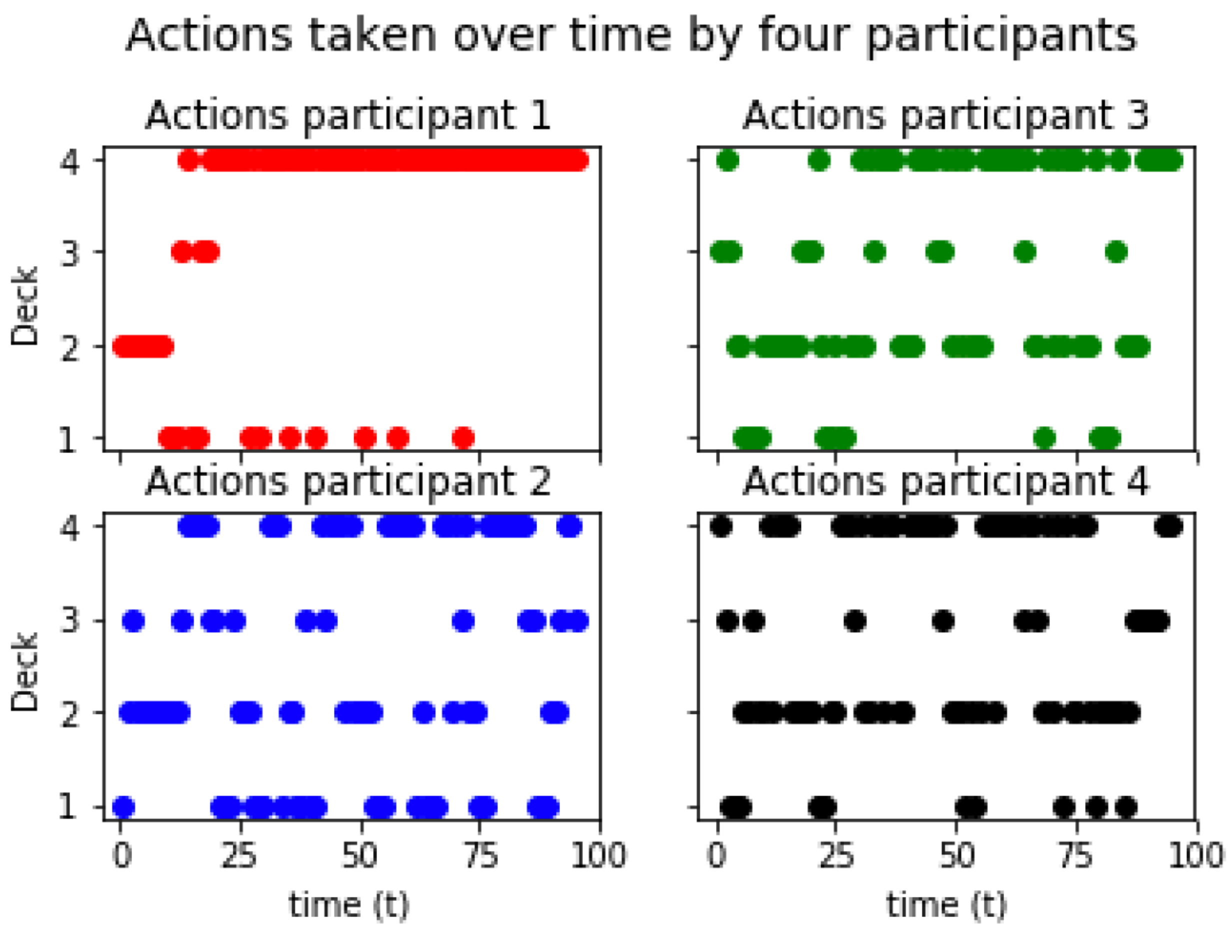
$$V(x) = \max_a \left[r(x,a) + \sum_y p(x,a,y)V(y) \right]$$



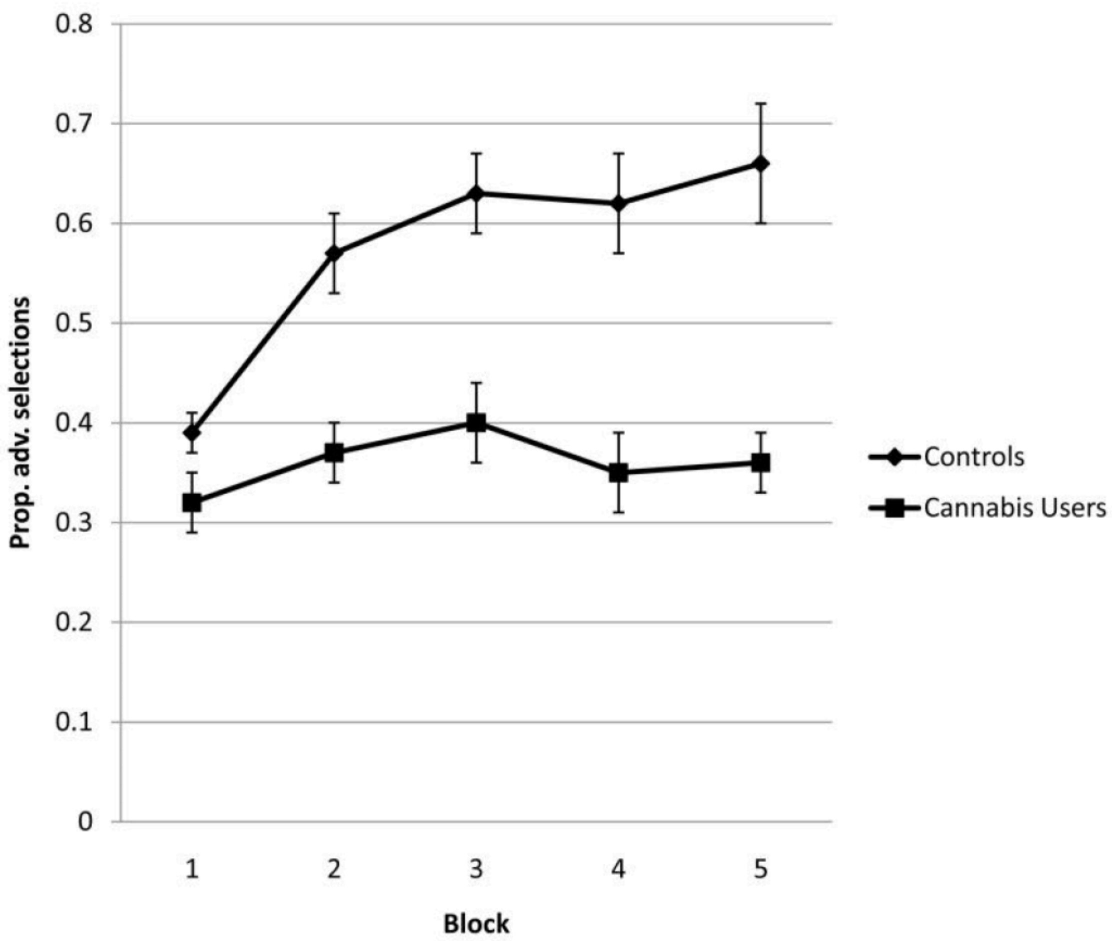
THE IOWA GAMBLING TASK

Demo

THE IOWA GAMBLING TASK



THE IOWA GAMBLING TASK



THE IOWA GAMBLING TASK

Stochastic Bandit setting

Environment: distributions $(\nu_1, \ldots \nu_K)$ of arm rewards.
Protocol: For $t = 1, 2, \ldots$

- Learner picks arm I_t
- Learner observes and receives *reward* $X_{I_t,t} \sim \nu_{I_t}$

Objective: Minimize pseudo-Regret w.r.t. best expert after T rounds:

$$\bar{R}_T = T\mu^* - \mathbb{E}_{I_1,\ldots,I_T}\left\{\sum_{t=1}^{T} X_{I_t,t}\right\}.$$

THE IOWA GAMBLING TASK

UCB algorithm

Protocol:

For $t = 1, \ldots, K$:

Initialize: $T_i(K) = 1, i = 1, 2, ..., K$.

For $t = K+1, K+2, \ldots, T$:

- Do:

$$I_t = \arg\max_{1 \le i \le K} \left[\hat{\mu}_{i,T_i(t-1)} + (\psi^*)^{-1}\left(\frac{\alpha \log t}{T_i(t-1)}\right)\right]$$

- Observe reward $X_{I_t,t}$

THE IOWA GAMBLING TASK

UCB algorithm for the IGT

Protocol:

For $t = 1, \ldots, 4$:

Initialize: $T_i(4) = 1, i = 1, 2, 3, 4$.

For $t = 5, 6, \ldots, 95$:

- Do:

$$I_t = \arg \max_{1 \leq i \leq K} \left[\hat{\mu}_{i,T_i(t-1)} + \sqrt{\frac{\alpha \log t}{2T_i(t-1)}} \right]$$

- Observe reward $X_{I_t,t}$

THE IOWA GAMBLING TASK

Q-learning for the IGT

Protocol:
Choose $\epsilon < 1$
Initialize for $a = 1, 2, 3, 4$:
$Q(a) \leftarrow 0$ and $N(a) \leftarrow 0$

- Loop until $t = T = 95$:

$$A = \begin{cases} \arg\max_a Q(a) & \text{w.p. } 1 - \epsilon \\ \sim \mathcal{U}\{1, 4\} & \text{w.p. } \epsilon \end{cases}$$

- Observe reward R
- $N(A) \leftarrow N(A) + 1$
- $Q(A) \leftarrow Q(A) + \frac{1}{N(A)}\Big(R - Q(A)\Big)$

THE IOWA GAMBLING TASK

?

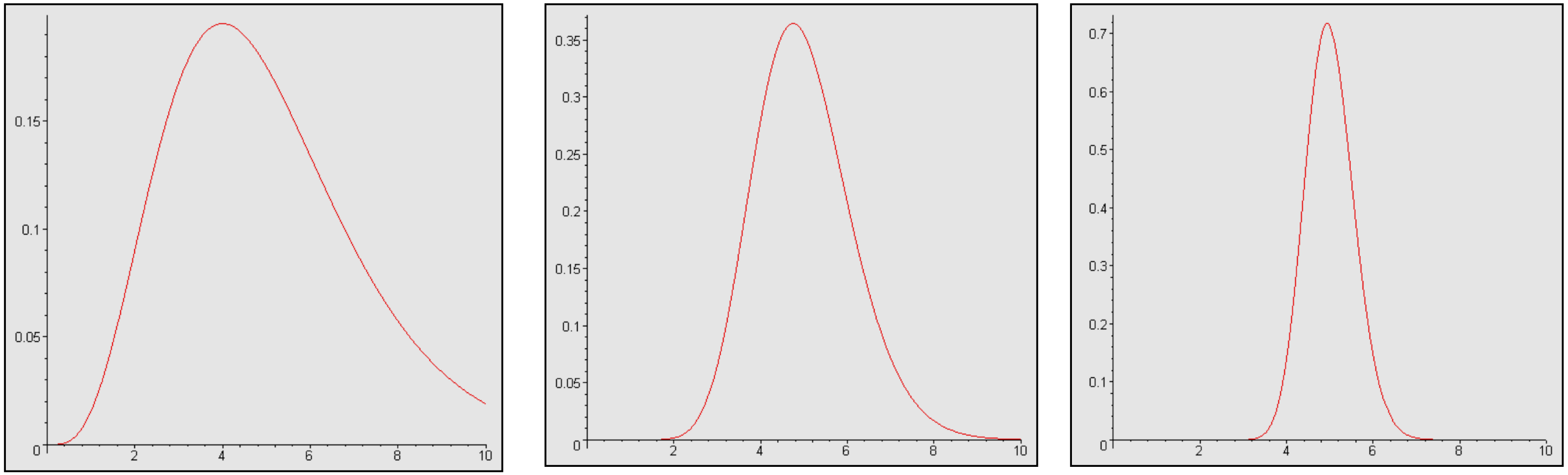
success probability p

success probability ???

THE IOWA GAMBLING TASK

- Tension between control vs. learning

Adaptive control



THE IOWA GAMBLING TASK

Thompson sampling for the IGT

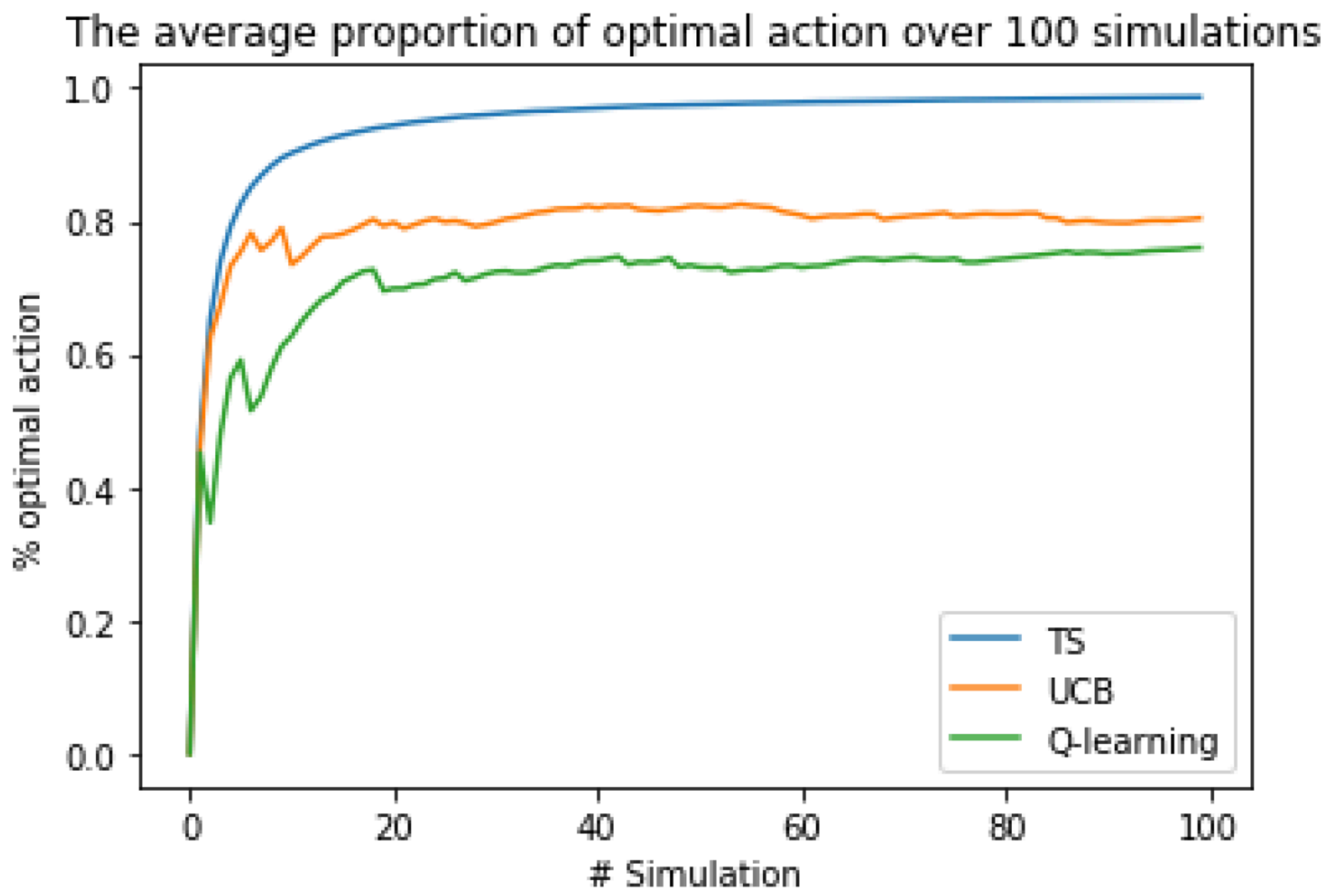
Protocol:
For each $\boldsymbol{\theta_i}$ with $i \in \{1, 2, 3, 4\}$ set

- $\boldsymbol{\theta_i} \sim \text{Dir}(\boldsymbol{\alpha_i})$

as prior. Where $\boldsymbol{\alpha_1} = (1, 1, 1, 1, 1, 1)$, $\boldsymbol{\alpha_2} = (1, 1)$, $\boldsymbol{\alpha_3} = (1, 1, 1, 1)$ and $\boldsymbol{\alpha_4} = (1, 1)$
For each $t = 1, \ldots, 95$ do:

- Draw a sample $\boldsymbol{\theta_i} \sim \text{Dir}(\boldsymbol{\alpha_i})$ for each $i \in \{1, 2, 3, 4\}$
- Compute $\mathbb{E}_{\boldsymbol{\theta_i}}[X_i]$ for each $i \in \{1, 2, 3, 4\}$
- $a_t = \arg\max_i \mathbb{E}_{\boldsymbol{\theta_i}}[X_i]$
- Observe outcome in deck i
- Update parameter $\boldsymbol{\alpha_i}$

THE IOWA GAMBLING TASK



REINFORCEMENT LEARNING

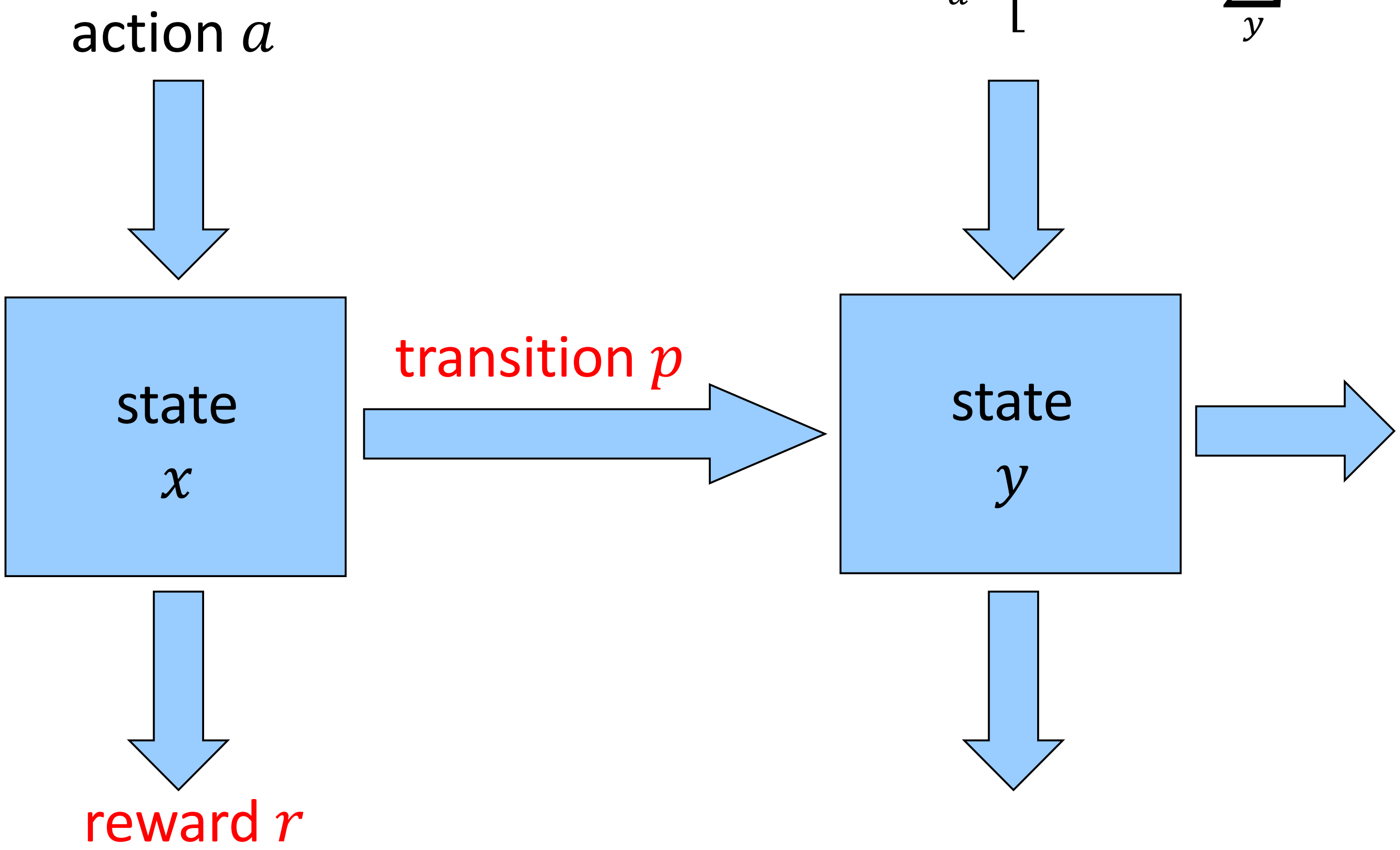
- Q-learning

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha \left(r_t + \gamma \max_{a'} Q(s_{t+1}, a') - Q(s_t, a_t) \right)$$

- Challenges:
  - The number of states might be very large
  - The state space might be very complex

MARKOV DECISION PROCESSES

$$V(x) = \max_a \left[r(x,a) + \sum_y p(x,a,y) V(y) \right]$$



REINFORCEMENT LEARNING

- Solution is function approximation

Model: $Q_\theta(s_t, a_t)$

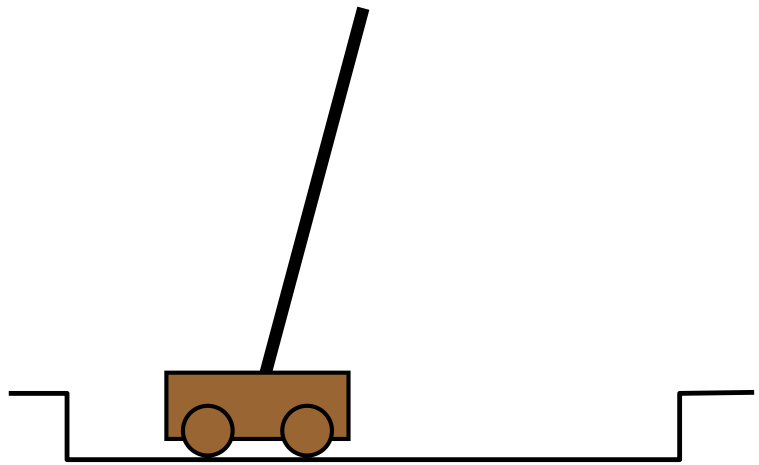
Training data: $\langle s_t, a_t, r_t, s_{t+1} \rangle$

Loss function: $\mathcal{L}(\theta) = ||y_t - Q_\theta(s_t, a_t)||_2^2$

where $y_t = r_t + \gamma Q(s_{t+1}, \pi(s_{t+1}))$

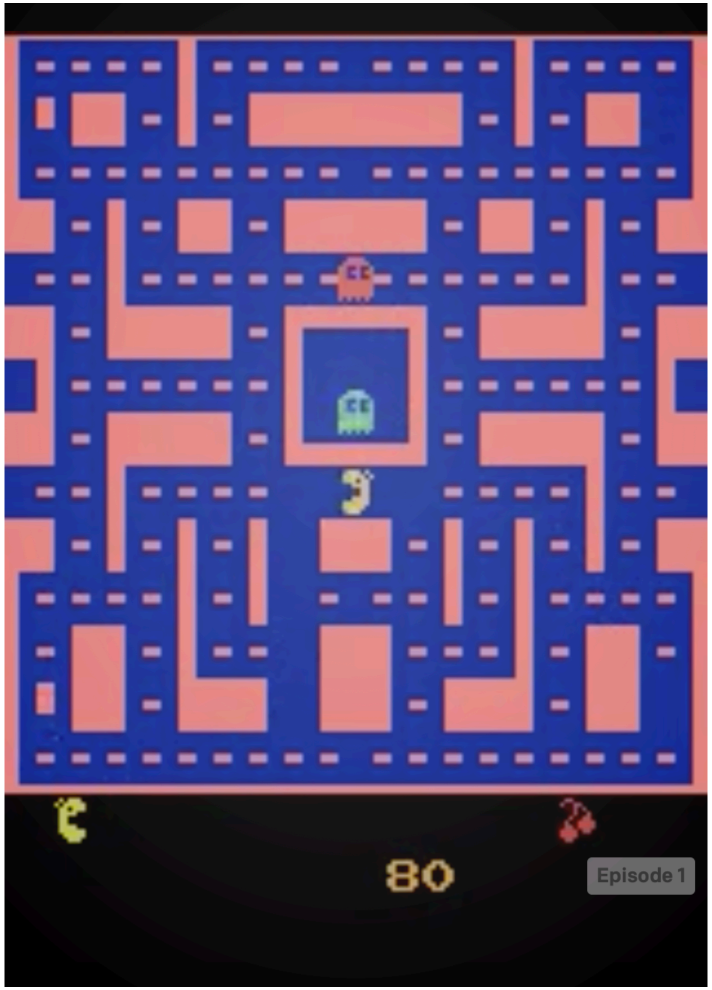
REINFORCEMENT LEARNING

- Pole-balancing
  - move car left/right to keep the pole balanced



- State representation
  - position and velocity of car
  - angle and angular velocity of pole
- Solution
  - coarse discretization of 4 state variables
  - left, center, right
  - totally non-Markov, but still works

OPEN AI



AMAZON DEEP RACER

QUESTIONS