AUDIO SIGNAL PROCESSING FOR SURVEILLANCE APPLICATIONS

Valencia, October 2007

Presented by

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CONTEXT

Group: Grupo de Tratamiento de Señal (**Signal Processing Group**)

Institution: Institute of Telecommunicacion and Multimedia Applications, Universidad Politécnica de Valencia **(Polytechnic University of Valencia),** SPAIN.

Members: 11 doctors, 14 undergraduate and postgraduate students/researchers.

Expertise:

Theory:

- signal detection and classification, time-frequency, nonlinear signal processing, higher-order statistics, image morphology algorithms, independent component analysis,…

Applications:

- Non-destructive testing of materials: ultrasonic and impact methods
- Surveillance systems: video, infrared and audio.
- Different applications of video analysis

CONTEXT

- ✓HESPERIA: Homeland sEcurity: tecnologíaS Para la sEguridad integRal en especios públicos e infrAestructuras. 2006-08. CENIT Programme.

- ✓Early warning of forest fires based on infrared signal processing. Special Programme of Valencia Government.

INDEX

- ✓Surveillance based on audio signals
- ✓Statistical signal processing
- ✓Experiences and demos

SURVEILLANCE BASED ON AUDIO SIGNAL

Why surveillance based on audio signals?

✓Sound gets information **not accessible from video**

- Hidden targets
- Surveillance in dark sites
- Abnormal sounds in normal images

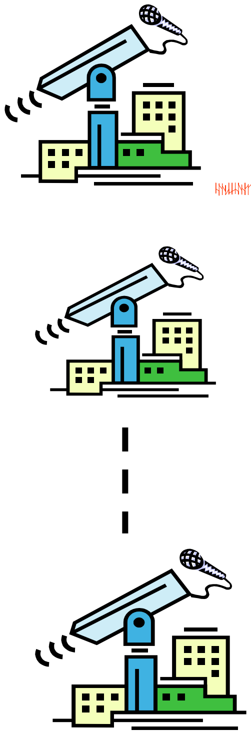
✓Sound gets information to be **combined with video** information

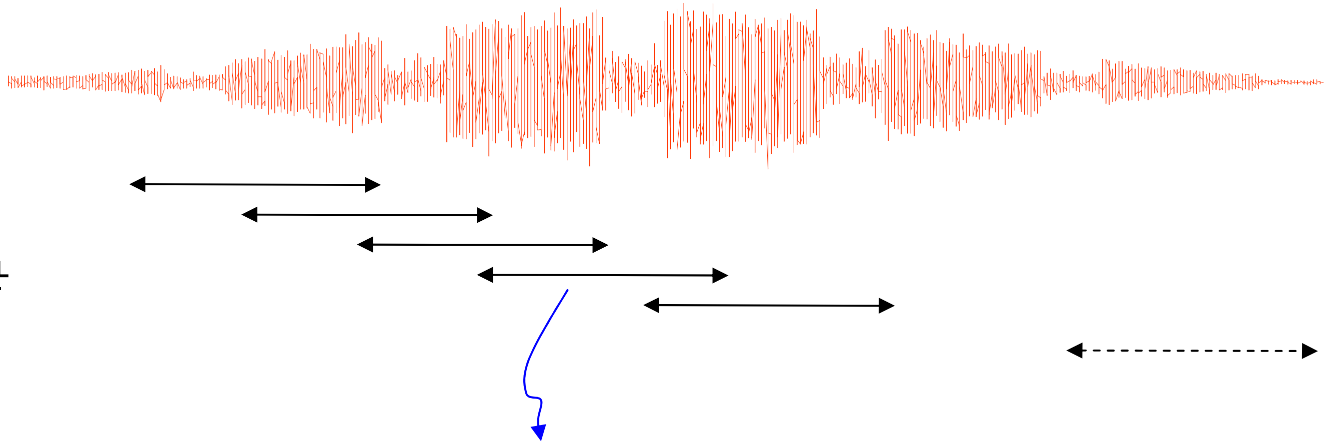
- Improved performance by decision/classification fusion
- Idex of video stream
- Steering of video camera to "interesting" directions

✓Sounds (except when voices are involved) achieves larger degree of **privacy** in comparison with video

SURVEILLANCE BASED ON AUDIO SIGNAL

Essential procedure





Observation vector **x**

Something new over the background?, if yes, **what** is it? And **where** is it?

SURVEILLANCE BASED ON AUDIO SIGNAL

Some examples of sound based surveillance

✓Surveillance at elevators

Radhakrishnan, R.; Divakaran, A., "Systematic Acquisition of Audio Classes for Elevator Surveillance", *SPIE Image and Video Communications and Processing*, Vol. 5685, pp. 64-71, March 2005.

- *8 sound classes in elevators are learned from 61 clips (1 hour) of suspicious events and 4 clips (40 minutes) without events:*
  - *Alarm, banging, elevator background, door opening & closing, elevator bell, footsteps, non-neutral speech, normal speech.*
- *A two step procedure was implemented:* ***detection*** *plus clustering*
- *Once learned a supervised* ***classifier*** *was designed to clasify every* ***1 sec*** *of sound.*

SURVEILLANCE BASED ON AUDIO SIGNAL

Some examples of sound based surveillance

✓Surveillance at home

Istrate et al.: "Remote monitoring sound system for distress situations detection,", *Innovation et technologie en biologie et médecine* ,ITBM-RBM, 2006.

- *Telesurveillance at home for elderly people and/or chronic patients.*
- *Different sensors combined with 1 microphone per room.*
- *Analyis window **32 ms.***
- ***Detection** followed by **classification** in 7 predefined classes of sounds:*

  Door closing, glass breaking, telephone sound, footsteps, scream, dishes, locks
- *Training+verfication based on 3354 files (3 hours)*

SURVEILLANCE BASED ON AUDIO SIGNAL

Some examples of sound based surveillance

✓Surveillance at office

Härma, M. F. McKinney, J. Skowronek: "Automatic Surveillance of the Acoustic Activity in our Living Environment", Proc. IEEE International Conference on Multimedia and Expo., Amsterdam, July 2005.

- *Continous leraning of interesting sounds in office environment*
- *Analysis window **62 ms**.*
- ***Detection** followed by **classification** in 25 subjectively selected classes of sounds:*

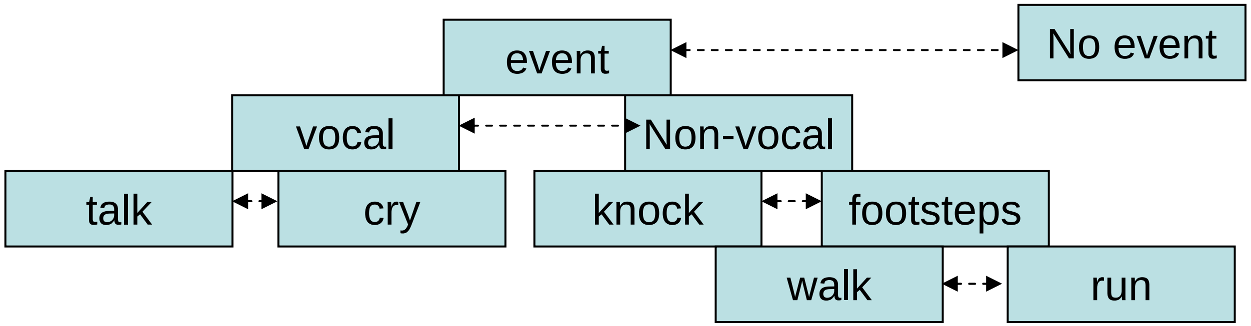
  Door closing, noise from different devices, voices,…
- *140.000 interesting events were recorded during 2 months*

SURVEILLANCE BASED ON AUDIO SIGNAL

Some examples of sound based surveillance

P. K. Atrey et al.: "Audio Based Event Detection for Multimedia Surveillance", Proc. IEEE International Conference on Acoustics, Speech and Signal Processing, Toulouse (France), May 2006.

- *Classification of sounds at office corridor into 6 categories*
- *A hierarchical approach is proposed:*



- *Analysis window 50 ms*
- *10 min of each even for training, 2 hours of recording for testing*

SURVEILLANCE BASED ON AUDIO SIGNAL

Some examples of sound based surveillance

✓Surveillance for intruder detection

•E. Menegatti, E. Mumolo, M. Nolich, E. Pagello "A Surveillance System based on Audio and Video Sensory Agents cooperating with a Mobile Robot" Proc. of 8th International Conference on Intelligent Autonomous Systems (IAS-8), March 2004, Amsterdam HOLANDA pp. 335-343

- *Audio+vision surveillance of the storage room of a shipping company*
- *6 microphone arrays (4 elements)+1 static omnidirectional camera+1 mobile robot with a camera*
- *Initial detection is made by the static camera, then the arrays steer towards the intruder and* ***track*** *it from the footsteps sound.*
- *Information on the* ***location*** *of the intruder is sent to the mobile robot to persue the intruder*

SURVEILLANCE BASED ON AUDIO SIGNAL

Some examples of sound based surveillance

✓Surveillance at the city

G. Valenzise et al.: "Scream and Gunshot Detection and Localization for Audio-Surveillance Systems", Proc. IEEE International Conference on Advanced Video and Signal based Surveillanace, London, September 2007.

- *A **classifier** is implemented to distinguish gunshots from background noise or scream from background noise*
- *Then the sound source is **localized** by array processing and a camera is steered towards it.*
- *Analyis window **23 ms.***
- *Training+verfication based on movie records, internet clips and recordings from a public square at Milan*

SURVEILLANCE BASED ON AUDIO SIGNAL

Some examples of sound based surveillance

C. Clavel et al.: "Events detection for an audio-based surveillance system", Proc. IEEE International Conference on Multimedia and Expo., Amsterdam, July 2005

- *Performs shot detection versus normality*
- *Analyis window **20 ms,** but a final decision is made every 0.5 sec.*
- *Background has been recorded in public spaces, then shots from a CD of radio recordings have been added*
- *Training+verfication besed on 134 shots (296 s) and 797 segments of normal noise of public places*

SURVEILLANCE BASED ON AUDIO SIGNAL

Some examples of sound based surveillance

✓Surveillance at railway stations

W. Zajdel et al.: “CASSANDRA: Audio-video Sensor Fusion for Aggression Detection”, Proc. IEEE International Conference on Advanced Video and Signal based Surveillanace, London, September 2007.

- *Analysis of voices in aggression condition, oriented to railway stations*
- *Fusion between audio and video is made from a probabilistic model and bayesian inference*
- *Presence of noise of trains is detected from video information to avoid false alarms*
- *13 clips (100 to 150 sec.) with actors in a railway station were recorded to test the system*

SURVEILLANCE BASED ON AUDIO SIGNAL

Some conclusions from the examples

✓There is a variety of scenarios and objectives

✓The analysis window length is somewhat arbitrary

✓**Novelty detection** is a key aspect in all cases

✓We may require **non-supervised** (learning the environment by clustering) and **supervised** (predefined classes of sounds) **classification**

✓**Localization** is sometimes required

SURVEILLANCE BASED ON AUDIO SIGNAL

What main difficulties do we have?

✓**Complex and variant background** producing **high intensity** sounds which should not confuse the sound surveillance system

✓How we **design/train the system** to satisfy some quality requirements?

SURVEILLANCE BASED ON AUDIO SIGNAL

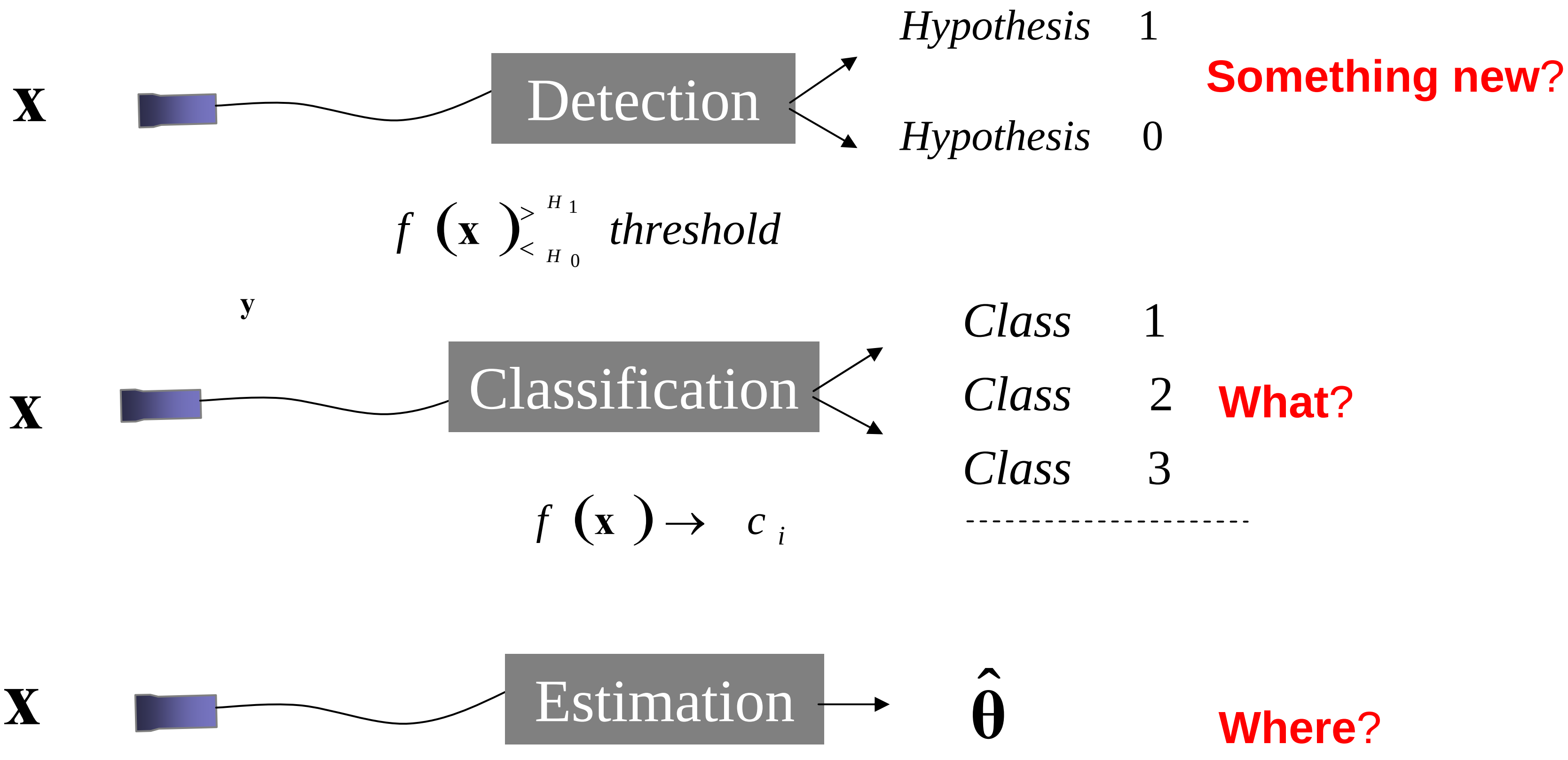
What help do we need?

✓**Statistical signal processing** allows the design of algorithms satisfying optimization criteria in a large variety of statistical models of the involved signals

- Control of probability of false alarm while maximizing probability of detection
- Selection of the most probable kind of sound
- Minimization of location/tracking errors

STATISTICAL SIGNAL PROCESSING

Given observation vector **x**, what is the optimum processing function $f(\mathbf{x})$?



$$f(\mathbf{x}) \underset{H_0}{\overset{H_1}{\gtrless}} \textit{threshold}$$

$$f(\mathbf{x}) \rightarrow c_i$$

$$\hat{\theta} = f(\mathbf{x})$$

STATISTICAL SIGNAL PROCESSING

Optimum detector: general

$$f(\mathbf{x}) = \frac{P(\mathbf{x}/H_1)}{P(\mathbf{x}/H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} threshold = \lambda$$

$P(\mathbf{x}/H_i)$ Is the probability density function (PDF) of **y** conditioned to H_i is the true hypothesis

$f(\mathbf{x})$ Is called the likelihood ratio

λ Is selected to fit a probability of false alarm (*PFA*) then probability of detection (*PD*) is maximized

Something new over the background?, if yes, **what** is it? And **where** is it?

STATISTICAL SIGNAL PROCESSING

Optimum detector: known signal in white Gaussian noise background

Matched filter

$$H_0: \quad \mathbf{x} = \mathbf{w} \qquad \mathbf{w} : N\left[\mathbf{0}, \sigma^2 \mathbf{I}\right]$$

$$H_1: \quad \mathbf{x} = a \cdot \mathbf{s} + \mathbf{w} \qquad \mathbf{s}^T \mathbf{s} = 1$$

$$f(\mathbf{x}) = \frac{\mathbf{s}^T \mathbf{x}}{\sigma} = z \underset{H_0}{\overset{H_1}{\gtrless}} \lambda \qquad \frac{1}{\sqrt{2\pi}} \int_{\lambda}^{\infty} \exp\left(-\frac{1}{2} z^2\right) dz = F(\lambda) = PFA$$

PDF of z is Gaussian

Problems:

- Noise must be **white Gaussian**
- Adaptive estimation of **noise level** σ is necessary
- A priori **knowledge of s** is required

STATISTICAL SIGNAL PROCESSING

Optimum detector: known signal in coloured Gaussian noise background

$$H_0: \quad \mathbf{x} = \mathbf{w} \qquad \mathbf{w} : N[\mathbf{0}, \mathbf{R}_w] \qquad \mathbf{R}_w = E[\mathbf{w}\mathbf{w}^T]$$

$$H_1: \quad \mathbf{x} = a \cdot \mathbf{s} + \mathbf{w} \qquad \mathbf{s}^T \mathbf{s} = 1$$

Prewhitened Matched filter

$$f(\mathbf{x}) = \frac{\mathbf{s}_w{}^T \mathbf{x}_w}{\sigma_w} = \frac{\left(\mathbf{R}_w^{-\frac{1}{2}} \mathbf{s}\right)^T \left(\mathbf{R}_w^{-\frac{1}{2}} \mathbf{x}\right)}{\sigma_w} = z \underset{H_0}{\overset{H_1}{\gtrless}} \lambda \qquad \frac{1}{\sqrt{2\pi}} \int_\lambda^\infty \exp\left(-\frac{1}{2} z^2\right) dz = F(\lambda) = PFA$$

PDF of z is Gaussian

Problems:

- Noise must be **coloured Gaussian**
- Adaptive estimation of **noise autocorrelation matrix $\mathbf{R}_\mathbf{w}$** is necessary
- A priori **knowledge of $\mathbf{s}$** is required

STATISTICAL SIGNAL PROCESSING

Optimum detector: unknown signal in white Gaussian noise background

$$H_0: \quad \mathbf{x} = \mathbf{w} \qquad \mathbf{w} : N\left[\mathbf{0}, \sigma^2 \mathbf{I}\right]$$

$$H_1: \quad \mathbf{x} = \mathbf{s} + \mathbf{w} \qquad \mathbf{s}\,?$$

Energy Detector

$$f(\mathbf{x}) = \frac{\mathbf{x}^T \mathbf{x}}{\sigma} = z \underset{H_0}{\overset{H_1}{\gtrless}} \lambda$$

$$\frac{1}{2^{\frac{N}{2}} \Gamma\left(\frac{N}{2}\right)} \int_{\lambda}^{\infty} z^{\frac{N}{2}-1} \exp\left(-\frac{1}{2} z\right) dz = F(\lambda) = PFA$$

PDF of z is Chi-square with N *(dimension of* $\mathbf{x}$*)* degrees of freedom

Problems:

- Noise must be **white Gaussian**
- Adaptive estimation of **noise level σ is required**

STATISTICAL SIGNAL PROCESSING

Optimum detector: unknown signal in coloured Gaussian noise background

$$H_0: \quad \mathbf{x}=\mathbf{w} \quad \mathbf{w}: N[\mathbf{0},\mathbf{R}_w]$$
$$H_1: \quad \mathbf{x}=\mathbf{s}+\mathbf{w} \quad \mathbf{s}?$$

Prewhitened Energy Detector

$$f(\mathbf{x})=\frac{\mathbf{x}_w^T\mathbf{x}_w}{\sigma_w}=\frac{\left(\mathbf{R}_w^{-\frac{1}{2}}\mathbf{x}\right)^T\left(\mathbf{R}_w^{-\frac{1}{2}}\mathbf{x}\right)}{\sigma_w}=z\underset{H_0}{\overset{H_1}{\gtrless}}\lambda$$

$$\frac{1}{2^{\frac{N}{2}}\Gamma\left(\frac{N}{2}\right)}\int_\lambda^\infty z^{\frac{N}{2}-1}\exp\left(-\frac{1}{2}z\right)dz=F(\lambda)=PFA$$

PDF of z is Chi-square with N *(dimension of* $\mathbf{x}$*)* degrees of freedom

Problems:

- Noise must be **coloured Gaussian**
- Adaptive estimation of **noise autocorrelation matrix $\mathbf{R}_\mathbf{w}$** is necessary

STATISTICAL SIGNAL PROCESSING

Optimum detector: unknown signal in coloured nonGaussian noise background

Preprocessed Energy Detector

$$H_0: \quad \mathbf{x}=\mathbf{w} \qquad \mathbf{w}?$$
$$H_1: \quad \mathbf{x}=\mathbf{s}+\mathbf{w} \qquad \mathbf{s}?$$

$$f(\mathbf{x})=\frac{\mathbf{x}_{wg}{}^T\mathbf{x}_{wg}}{\sigma_{wg}}=\frac{g\left(\mathbf{Q}\mathbf{R}_w^{-\frac{1}{2}}\mathbf{x}\right)^T g\left(\mathbf{Q}\mathbf{R}_w^{-\frac{1}{2}}\mathbf{x}\right)}{\sigma_{wg}}=z \underset{H_0}{\overset{H_1}{\gtrless}} \lambda \qquad \frac{1}{2^{\frac{N}{2}}\Gamma\left(\frac{N}{2}\right)}\int_\lambda^\infty z^{\frac{N}{2}-1}\exp\left(-\frac{1}{2}z\right)dz=F(\lambda)=PFA$$

PDF of z is Chi-square with N *(dimension of* $\mathbf{x}$*)* degrees of freedom

Problems:

- Adaptive estimation of **noise autocorrelation matrix $\mathbf{R}_\mathbf{w}$** is necessary
- Adaptive estimation of **rotation matrix $\mathbf{Q}$** and nonlinear mapping $g(.)$ is necessary
- Adaptive estimation of (transformed) **noise level** σ_{wg} is necessary

STATISTICAL SIGNAL PROCESSING

Optimum detector: learning the parameters **off-line**

We have a **training set** of background noise observation vectors:

$$\mathbf{w}_l \qquad l = 1 \ldots\ldots L$$

$$\hat{\sigma}^2 = \frac{1}{N \cdot L} \sum_{l=1}^{L} \mathbf{w}_l^T \mathbf{w}_l$$

$$\hat{\mathbf{R}}_w = \frac{1}{L} \sum_{l=1}^{L} \mathbf{w}_l \mathbf{w}_l^T$$

$$\left.\begin{matrix} \hat{g} \\ \hat{\mathbf{Q}} \end{matrix}\right\} \quad \hat{\mathbf{Q}} = \frac{1}{L} \sum_{l=1}^{L} \hat{g}\left(\hat{\mathbf{Q}} \hat{\mathbf{R}}_w^{-\frac{1}{2}} \mathbf{w}_l \right) \mathbf{w}_l^T \hat{\mathbf{R}}_w^{-\frac{1}{2}}$$

→ Nonlinear equations, can be solved by iterative methods

$$\hat{\sigma}_{wg}^2 = \frac{1}{N \cdot L} \sum_{l=1}^{L} g\left(\mathbf{Q} \mathbf{R}_w^{-\frac{1}{2}} \mathbf{w}_l \right)^T g\left(\mathbf{Q} \mathbf{R}_w^{-\frac{1}{2}} \mathbf{w}_l \right)$$

STATISTICAL SIGNAL PROCESSING

Optimum detector: learning the parameters on-line

We update the parameters with every new observation vector where H_0 has been selected

$$\hat{\sigma}_{t+1}^2 = \alpha\hat{\sigma}_t^2 + (1-\alpha)\mathbf{w}_t^T\mathbf{w}_t$$

$$\hat{\mathbf{R}}_{w(t+1)} = \beta\hat{\mathbf{R}}_{w(t)} + (1-\beta)\mathbf{w}_n\mathbf{w}_n^T$$

$$\left.\begin{matrix}\hat{g}\\ \hat{\mathbf{Q}}\end{matrix}\right\} \quad \hat{\mathbf{Q}}_{t+1} = \frac{1}{M}\sum_{l=t-M}^{t}\hat{g}_{t+1}\left(\hat{\mathbf{Q}}_{t+1}\hat{\mathbf{R}}_{wt}^{-\frac{1}{2}}\mathbf{w}_l\right)\mathbf{w}_l^T\hat{\mathbf{R}}_{wt}^{-\frac{1}{2}}$$

→ Nonlinear equations, can be solved by iterative methods

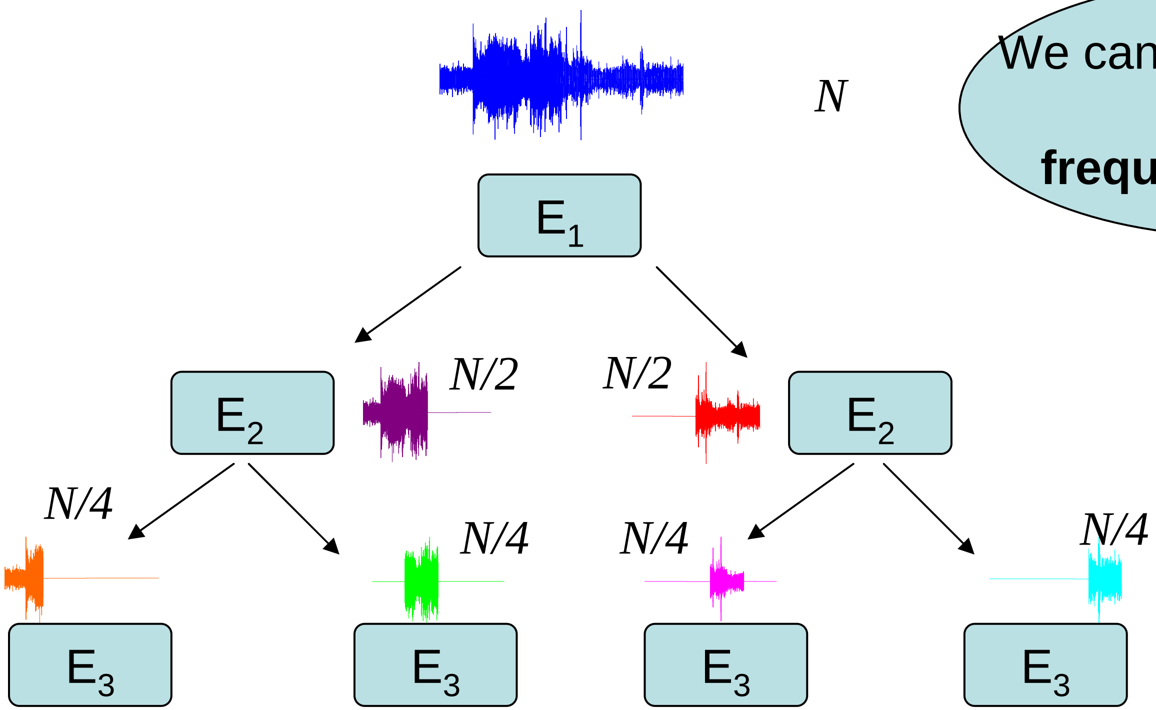
$$\hat{\sigma}_{wg(t+1)}^2 = \varepsilon\hat{\sigma}_{wg(t)}^2 + (1-\varepsilon)g\left(\hat{\mathbf{Q}}_t\hat{\mathbf{R}}_{w(t)}^{-\frac{1}{2}}\mathbf{w}_t\right)^T g\left(\hat{\mathbf{Q}}_t\hat{\mathbf{R}}_{w(t)}^{-\frac{1}{2}}\mathbf{w}_t\right)$$

STATISTICAL SIGNAL PROCESSING

Optimum detector: dealing with **unknown duration of the event**

We can try different values of dimension *N* to improve detectability:

A pyramid of energy detectors

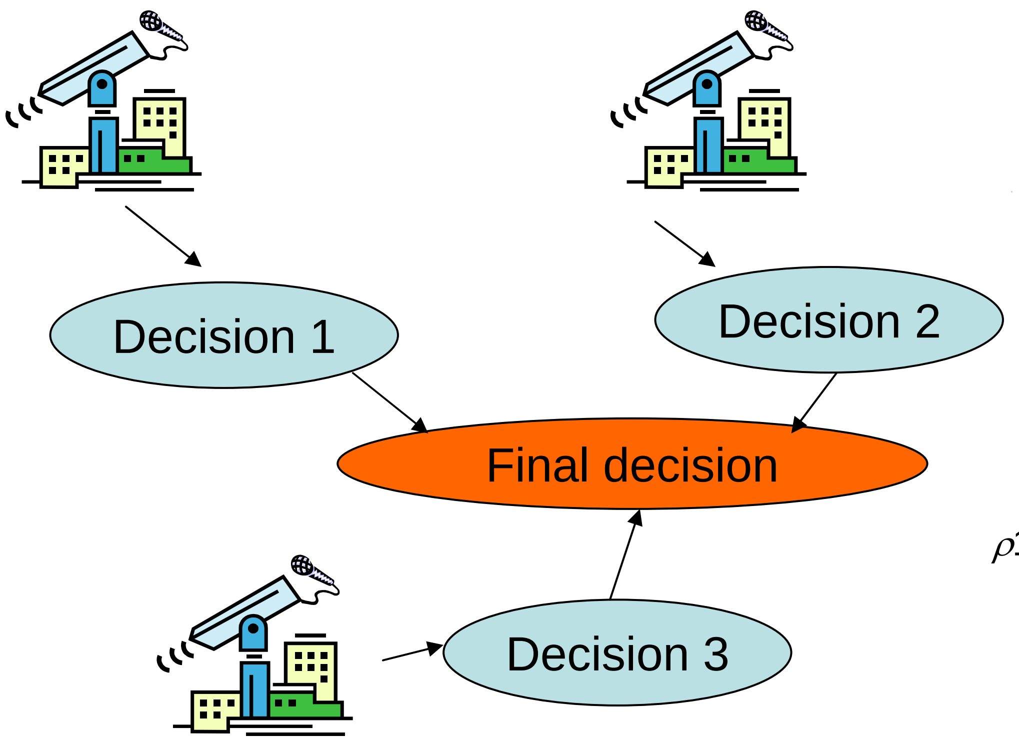


We can build pyramids also in **frequency domain**

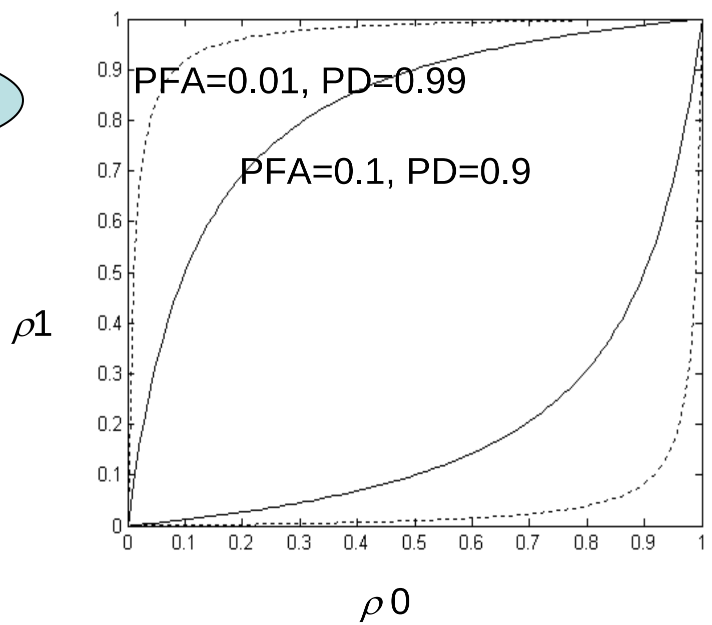
STATISTICAL SIGNAL PROCESSING

Optimum detector: dealing with **fusion of decisions** from different microphones

We can try different values of dimension *N* to improve detectability:



Counting rules have been proved to be optimum decision rule in most cases



STATISTICAL SIGNAL PROCESSING

Optimum detector: summary

✓**Energy detector** overcomes the unknowledge about signal waveform

✓**Prewhitening-rotation-nonlinear mapping** are required in the most general case before computing the energy

✓**Fitting the threshold** for a required PFA is an easy problem

✓**Learning the background** (i.e., training the detector) is a complex problem which should be very specific for every scenario/applicatio

✓**Frequency domain** can be very useful in some cases

✓**Decision fusion** is not complicated in general

STATISTICAL SIGNAL PROCESSING

Optimum classifier: why classify?

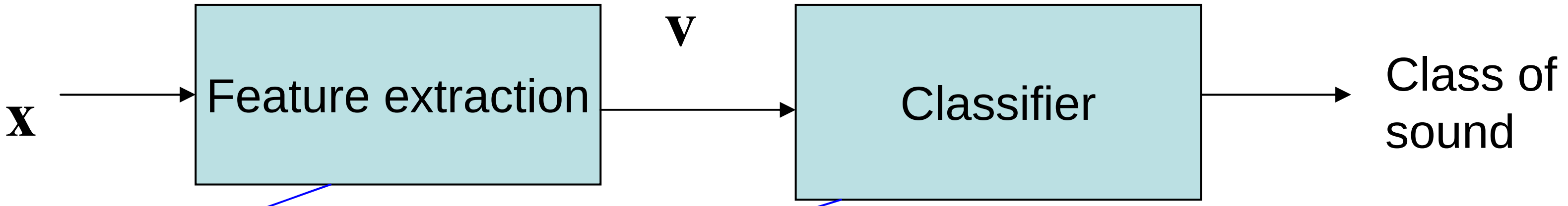
✓Once a detection is made, determining the kind of sound may be useful to:

- ➢Determine the degree of threat of the event
- ➢Help in establishing the best strategy against the threat

Something new over the background?, if yes, **what** is it? And **where** is it?

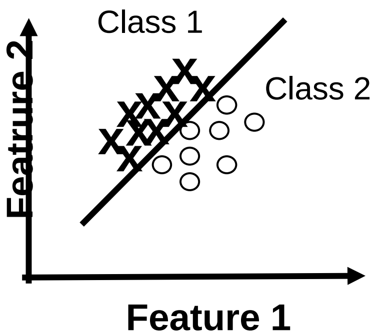
STATISTICAL SIGNAL PROCESSING

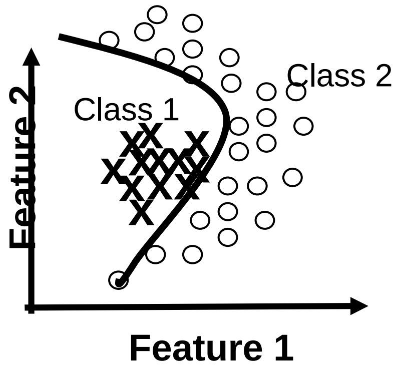
Optimum classifier: basic scheme



Proper feature selection is a key aspect for success

Many options, probably some of them are valid for every specific problem





STATISTICAL SIGNAL PROCESSING

Optimum classifier: general

✓The classifier should select the **most probable class** given the observed feature vector, then the key problem is to determine

$p(C_k / \mathbf{v})$ Then we select C_k *having* $\underbrace{\max p(C_{k'} / \mathbf{v})}_{C_{k'}}$

Using Bayes

$$p(C_k / \mathbf{v}) = \frac{p(\mathbf{v} / C_k) \cdot P(C_k)}{p(\mathbf{v})}$$

We have a problem of multivariate PDF estimation

STATISTICAL SIGNAL PROCESSING

Optimum classifier: a general model

Let us assume that the feacture vectors of class C_k can be expresses as

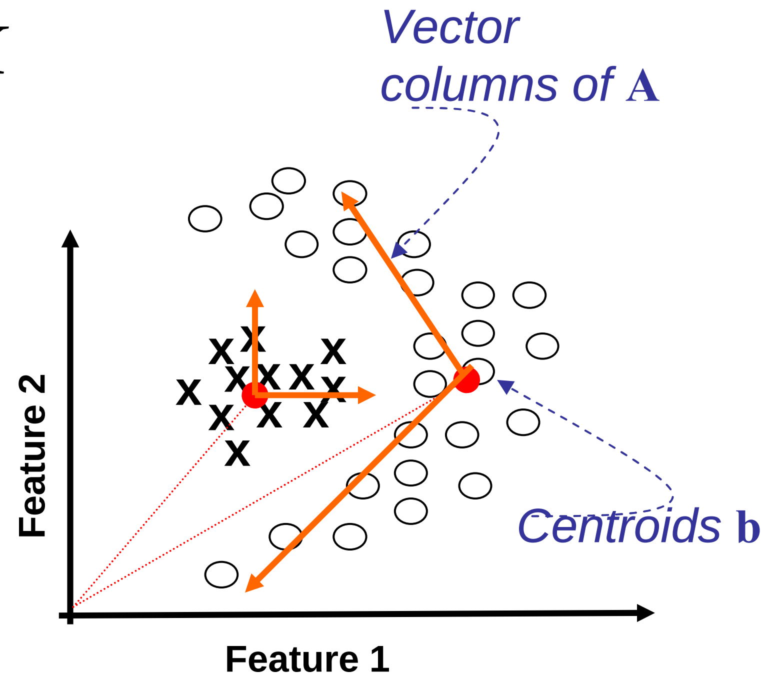
$$\mathbf{v}_k = \mathbf{A}_k \mathbf{s}_k + \mathbf{b}_k \qquad k = 1 \ldots K$$

Where $\mathbf{s}_k$ is a vector of **independent components**

$$p(\mathbf{s}_k) = p(s_1)p(s_2)\ldots p(s_K)$$

then

$$p(C_k / \mathbf{v}) = \frac{\left|\det \mathbf{A}_k^{-1}\right| p(\mathbf{s}_k)}{\sum_{k'=1}^{K} \left|\det \mathbf{A}_{k'}^{-1}\right| p(\mathbf{s}_{k'})} \qquad \mathbf{s}_k = \mathbf{A}_k^{-1}(\mathbf{v} - \mathbf{b}_k)$$



¿$\mathbf{A}_k$, $p(\mathbf{s}_k)$, $\mathbf{b}_k$, ?

STATISTICAL SIGNAL PROCESSING

Optimum classifier: Gaussian case

White case

$$\mathbf{v}_k = \mathbf{s}_k + \mathbf{b}_k \qquad k = 1 \ldots K$$

$$p(C_k / \mathbf{v}) \propto \left\| \mathbf{v} - \mathbf{b}_k \right\|^2 = (\mathbf{v} - \mathbf{b}_k)^T (\mathbf{v} - \mathbf{b}_k)$$

Coloured case

$$\mathbf{v}_k = \mathbf{R}_k^{\frac{1}{2}} \mathbf{s}_k + \mathbf{b}_k \qquad k = 1 \ldots K$$

$$p(C_k / \mathbf{v}) \propto \left\| \mathbf{R}_k^{-\frac{1}{2}} (\mathbf{v} - \mathbf{b}_k) \right\|^2 = (\mathbf{v} - \mathbf{b}_k)^T \mathbf{R}_k^{-1} (\mathbf{v} - \mathbf{b}_k)$$

STATISTICAL SIGNAL PROCESSING

Optimum classifier: Training (supervised)

$$\mathbf{v}_k^{(l)} \qquad l = 1 \ldots L_k, \quad k = 1 \ldots K$$

$$\hat{\mathbf{b}}_k = \frac{1}{L_k} \sum_{l=1}^{L_k} \mathbf{v}_k^{(l)}$$

$$\hat{\mathbf{R}}_k = \frac{1}{L_k} \sum_{l=1}^{L_k} \left(\mathbf{v}_k^{(l)} - \hat{\mathbf{b}}_k\right)\left(\mathbf{v}_k^{(l)} - \hat{\mathbf{b}}_k\right)^T$$

Extension to **unsupervised** training is straightforward

$$\left.\begin{array}{l}\mathbf{A}_k \\ p(\mathbf{s}_k)\end{array}\right\}$$

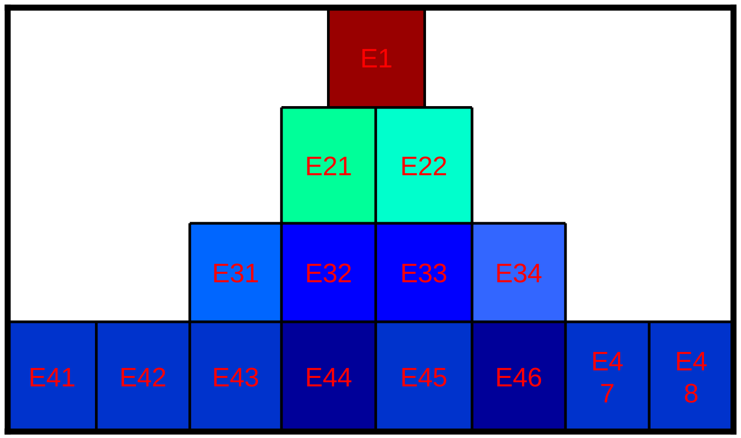
In the general nonGaussian case we can use Independent Component Analysis iterative algorithms to estimate the mixing matrix

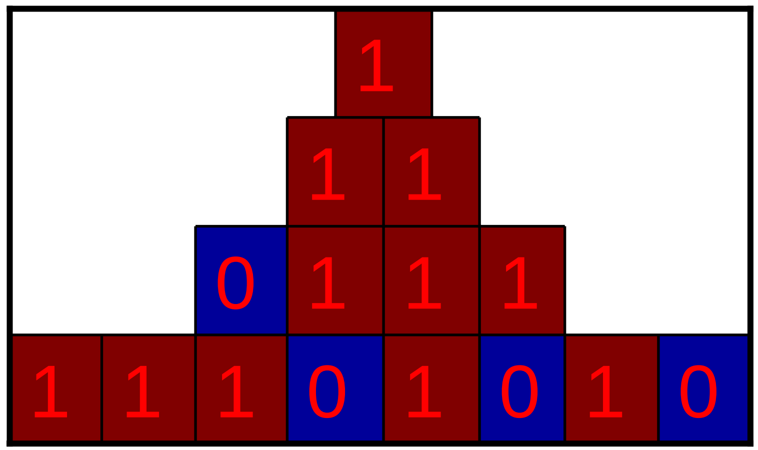
STATISTICAL SIGNAL PROCESSING

Optimum classifier: what about features ?

✓There is some trend to use features which are usual in speech analysis: cepstral parameters. There could be not appropriate for general kind of sounds.

✓Why not trying **energy features** coming from the pyramid analysis?



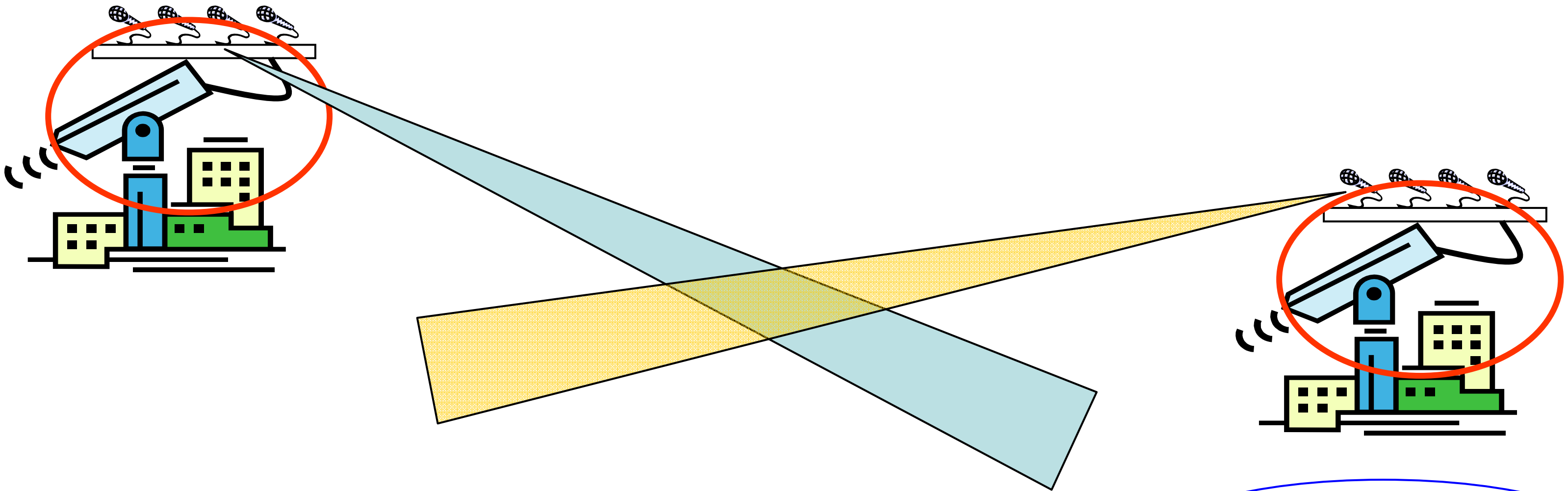


STATISTICAL SIGNAL PROCESSING

Optimum location: basics

✓We need an **array of microphones** instead of only one microphone

✓By **triangularization** we can locate the source of sound



Something new over the background?, if yes, **what** is it? And **where** is it?

STATISTICAL SIGNAL PROCESSING

Optimum location: DOA estimation I

✓Every array must estimate the direction of arrival of the sound source

✓This is a classical estimation problem

✓The optimum estimator should maximize the probability of the observation vector conditioned to the angle of arrival

$$P\left(\mathbf{x}/\gamma\right)$$

✓Solutions are well-known in the **Gaussian noise case**

STATISTICAL SIGNAL PROCESSING

Optimum location: DOA estimation I

$$\mathbf{x}_l = [x_{l1} \ldots x_{lM}]^T \qquad i = 1 \ldots L$$

Snapshots formed by the values received at the same time in every sensor

$$\hat{\gamma} = \underbrace{\max}_{\gamma} \left[\mathbf{a}(\gamma)^T \, \hat{\mathbf{R}} \, \mathbf{a}(\gamma) \right] \longrightarrow$$

Spacial energy detector

$$\mathbf{a} = \left[1 e^{-j2\pi \frac{d}{c} fsen\gamma} \ldots e^{-j2\pi \frac{dM}{c} fsen\gamma} \right]^T$$

$$\hat{\mathbf{R}} = \frac{1}{L} \sum_{l=1}^{L} \mathbf{x}_l \mathbf{x}_l^T$$

Narrowband case. The sanpshots are assumed to be composed by a spatial sinusoid in spatial white Gaussian noise

STATISTICAL SIGNAL PROCESSING

Optimum location: DOA estimation II

$$\mathbf{x}_l(f_i) = [x_{l1}(f_i) \ldots x_{lM}(f_i)]^T \qquad i = 1 \ldots L$$

Snapshots formed by the values of the Fourier transform of the received signals in every sensor

$$\hat{\gamma} = \frac{1}{I}\sum_{i=1}^{I} \underbrace{\max}_{\gamma}\left[\mathbf{a}_{f_i}(\gamma)^T \hat{\mathbf{R}}_{f_i} \mathbf{a}_{f_i}(\gamma)\right]$$

$$\mathbf{a}_{f_i}(\gamma) = \left[1 \; e^{-j2\pi \frac{d}{c} f_i sen\gamma} \ldots e^{-j2\pi \frac{dM}{c} f_i sen\gamma}\right]^T$$

$$\hat{\mathbf{R}}_{f_i} = \frac{1}{L}\sum_{l=1}^{L} \mathbf{x}_l(f_i)\mathbf{x}_l^T(f_i)$$

Wideband case. For every frequency, the sanpshots are assumed to be composed by a spatial sinusoid in spatial white Gaussian noise